C1 Exercise 2A (plotting quadratic graphs)

Note Title 02/08/2006

1a $y = x^2 - 3$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-3	-3	-3	-3	-3	-3	-3	-3	-3	-3
y	13	6	1	-2	-3	-2	1	6	13

This method of setting it out, suggested in the textbook (p14), is excellent. Take your time. Check your working. Show your method. These are easy marks don't miss 'em.

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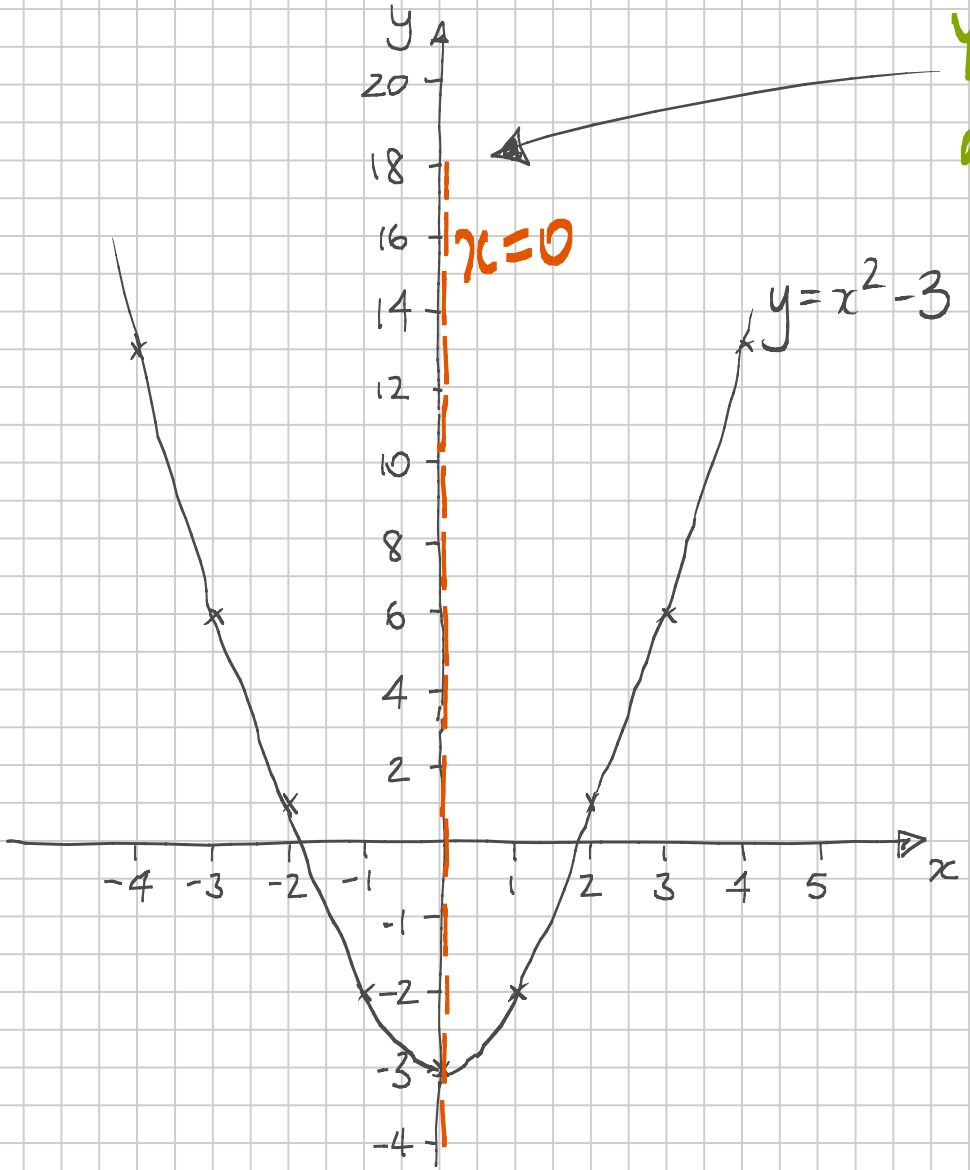


You too should be using a sharp pencil like me.

(But please use a ruler. I would, but I fear scratching the screen.)

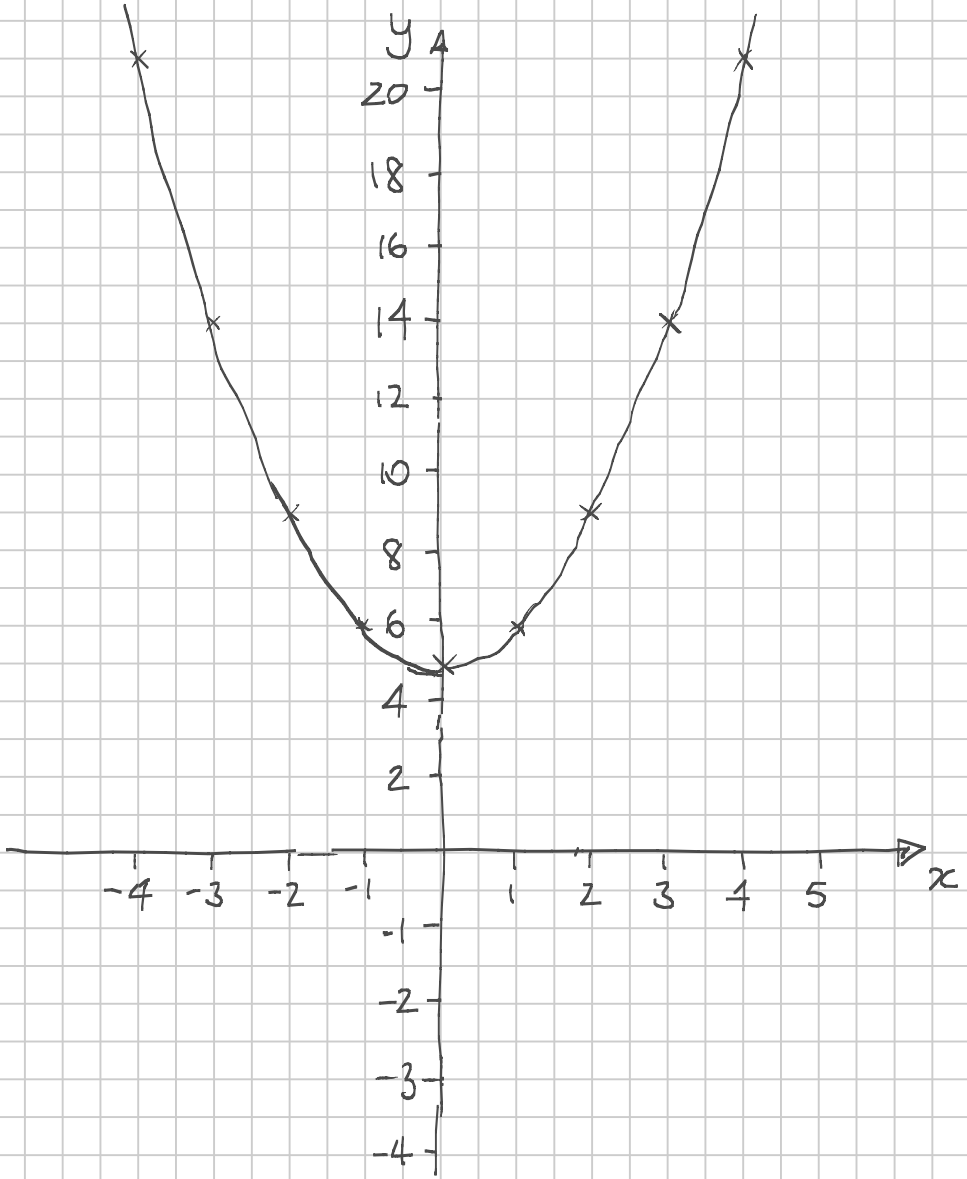
The line of symmetry is $x = 0$.

To get a smooth (ish) curve turn your paper upside down half way along the curve!

See p19 in this doc.

2a $y = x^2 + 5$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
+5	+5	+5	+5	+5	+5	+5	+5	+5	+5
y	21	14	9	6	5	6	9	14	21

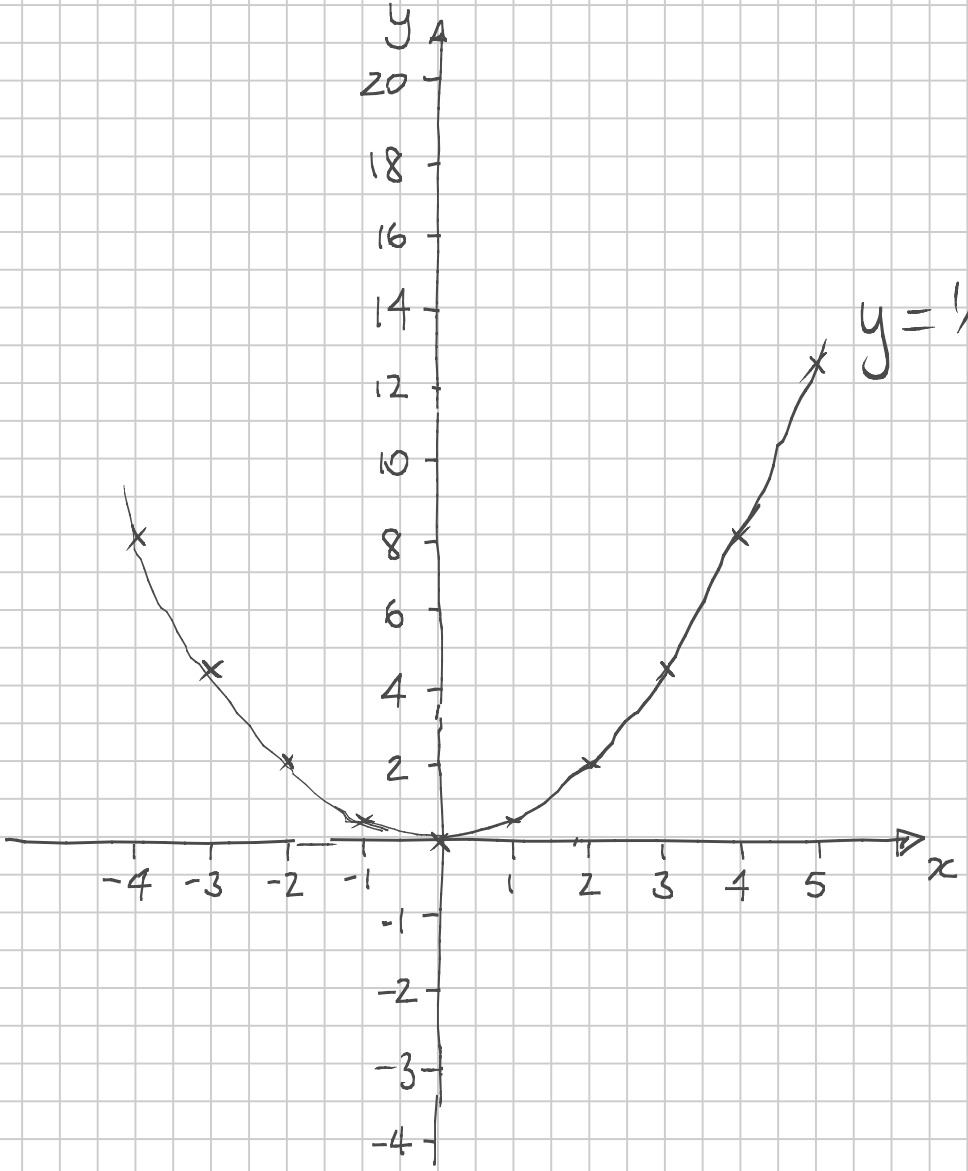


The line of symmetry is $x = 0$.
(again)

3a $y = \frac{1}{2}x^2$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$y=\frac{1}{2}x^2$	8	4½	2	½	0	½	2	4½	8

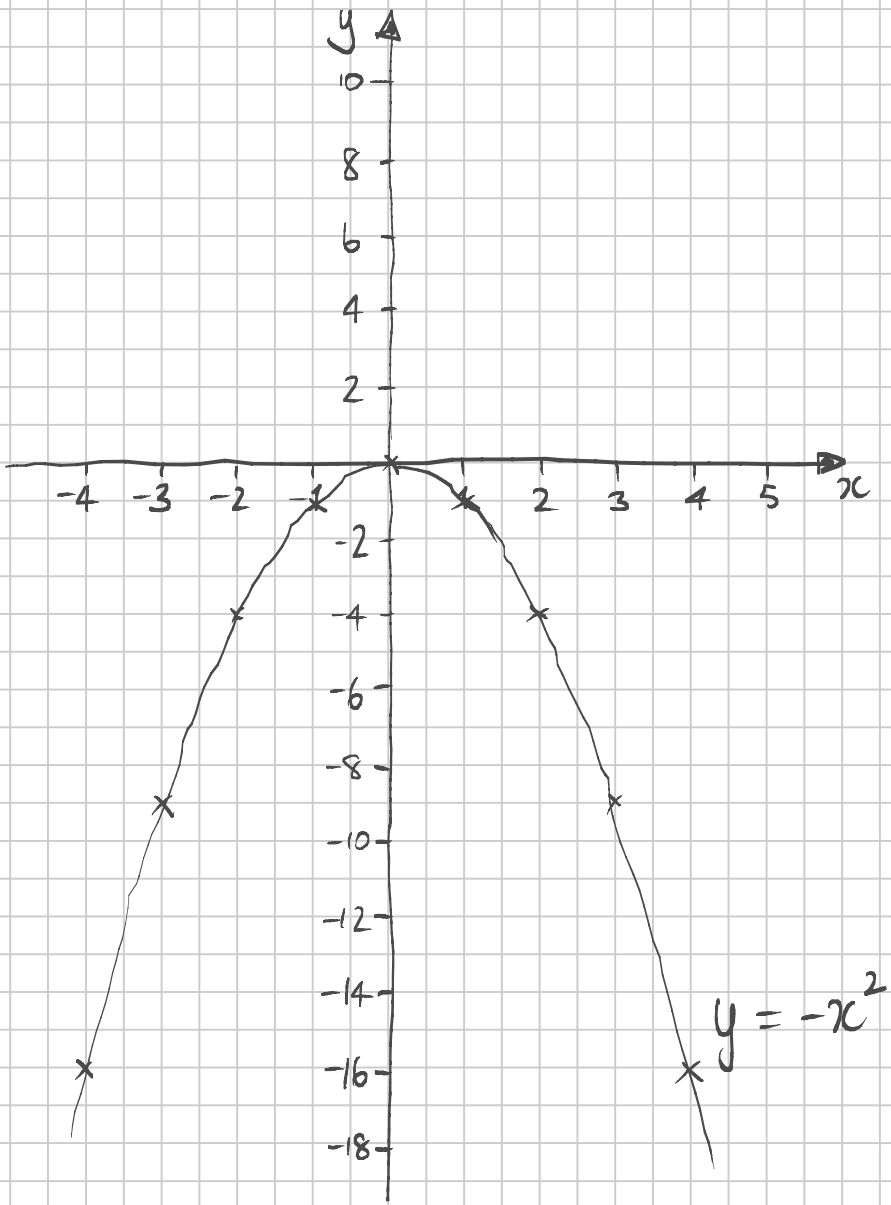
3b



The line of symmetry is still $x = 0$

4a $y = -x^2$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$y = -x^2$	-16	-9	-4	-1	0	-1	-4	-9	-16



Even when the coefficient of x^2 is negative we still get a line of symmetry at $x = 0$.

So what do our first four quadratics have in common that gives them this consistent line of symmetry?

Think about it...

5a $y = (x-1)^2$

x	-4	-3	-2	-1	0	1	2	3	4
$x-1$	-5	-4	-3	-2	-1	0	1	2	3
$y=(x-1)^2$	25	16	9	4	1	0	1	4	9

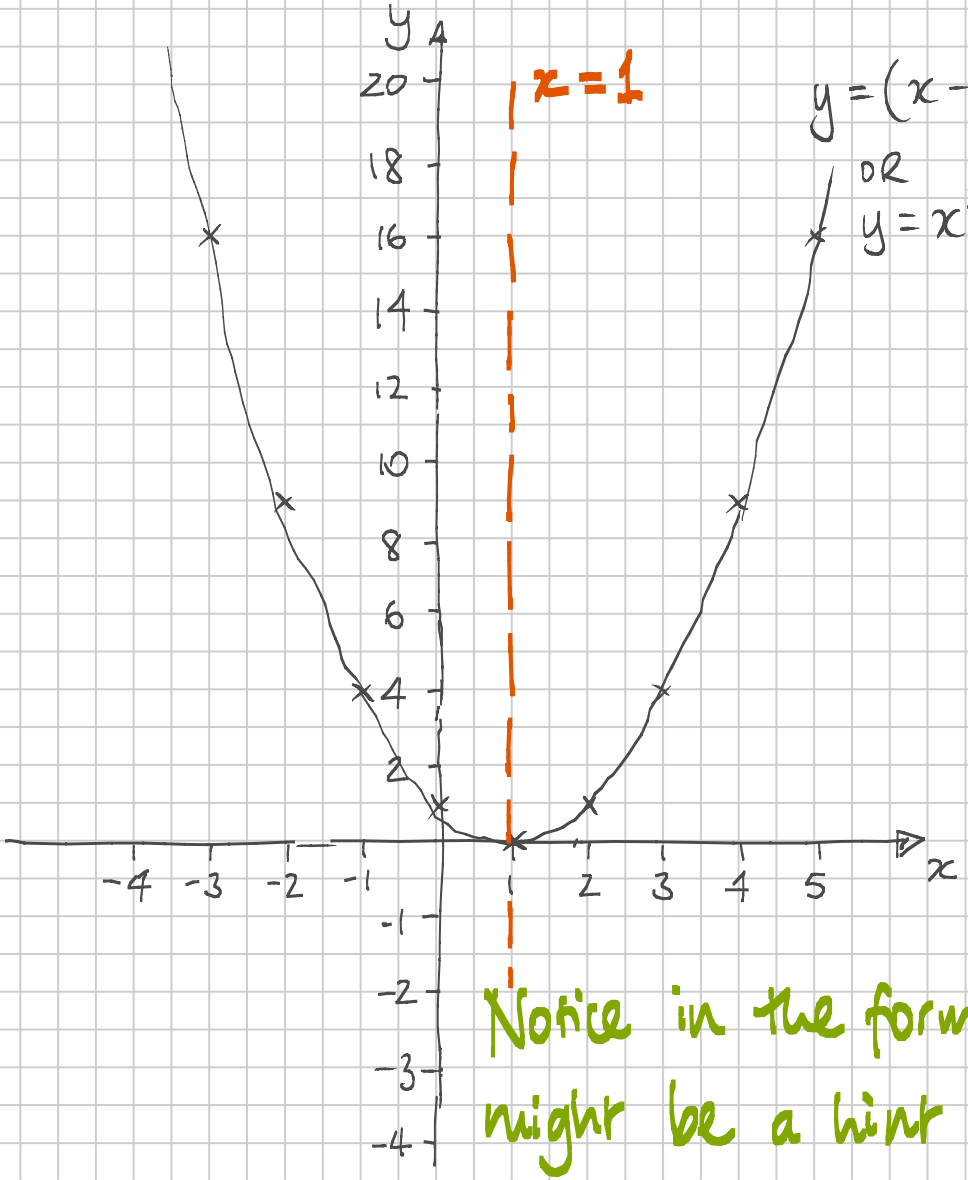
Does it strike you as a bit odd to have jumped to quadratics with brackets in? What's that all about?

Lets re-write $y = (x-1)^2$ in a more 'normal' way:

$(x-1)(x-1) = x^2 - 2x + 1$.

Keep that new equation in mind as we plot these results.

5b



Now the line of symmetry is at $x = 1$.

And in the 'standard' way of writing it, we're plotting

$$y = x^2 - 2x + 1$$

This is what's new: there's a term with x in it.

Notice in the form $y = (x - 1)^2$ there might be a hint about the line of symmetry.

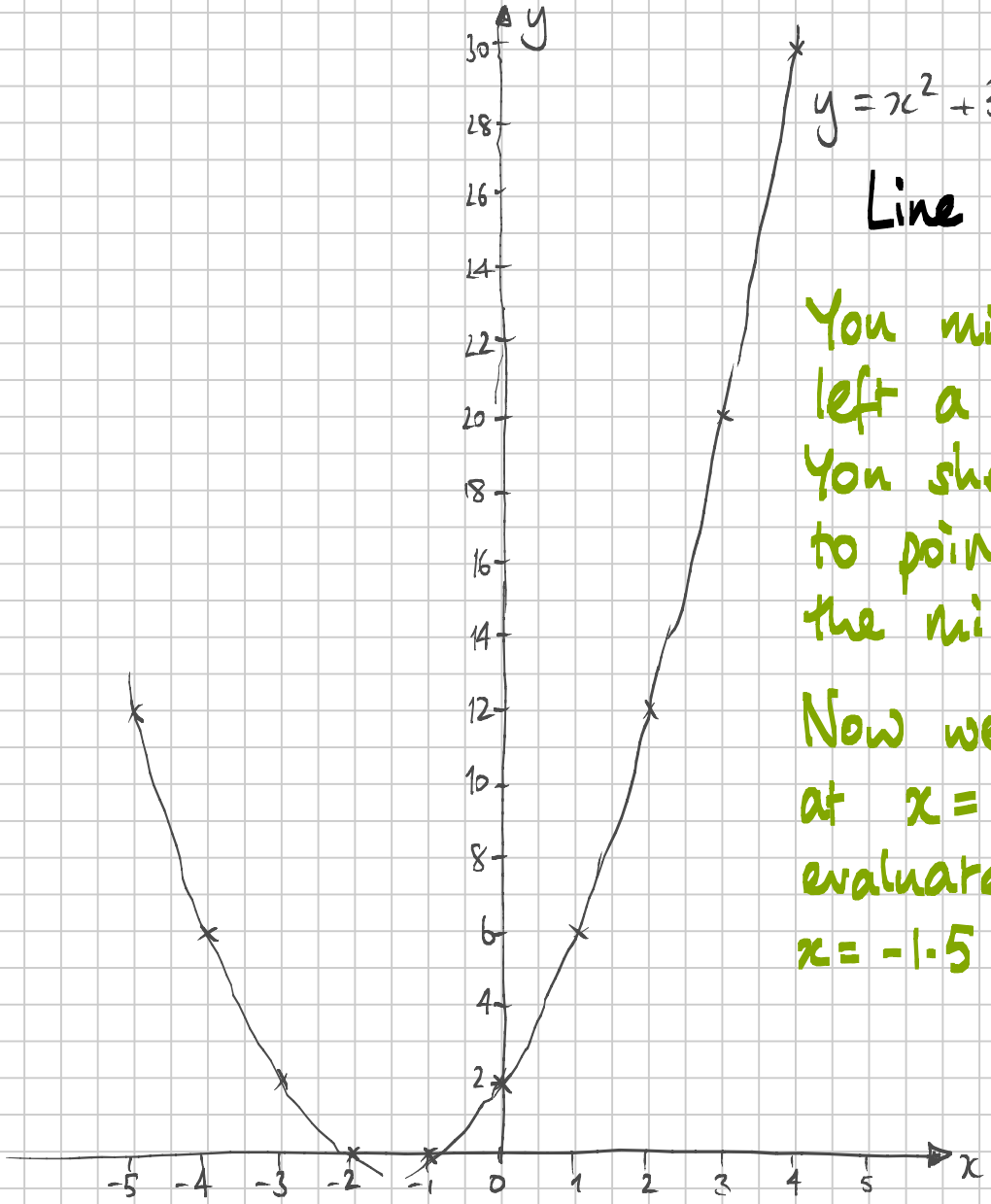
6a $y = x^2 + 3x + 2$

This needs a more subtle table to get the three terms. See the example on page 14 of the C1 textbook.

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$+3x$	-12	-9	-6	-3	$+0$	$+3$	$+6$	$+9$	$+12$
$+2$	$+2$	$+2$	$+2$	$+2$	$+2$	$+2$	$+2$	$+2$	$+2$
y	6	2	0	0	2	6	12	20	30

Watch out for the minus signs here especially.

6b



$y = x^2 + 3x + 2$

Line of symmetry $x = -1\frac{1}{2}$.

You might have noticed I've left a gap at around $x = -1\frac{1}{2}$. You shouldn't, but I wanted to point out we don't yet know the minimum value for y.

Now we're pretty sure it lies at $x = -1\frac{1}{2}$ why don't we evaluate and find it?

$x = -1.5 \Rightarrow x^2 = 2.25,$

$3x = -4.5$

$+2 = +2$

$\Rightarrow y = -0.25.$

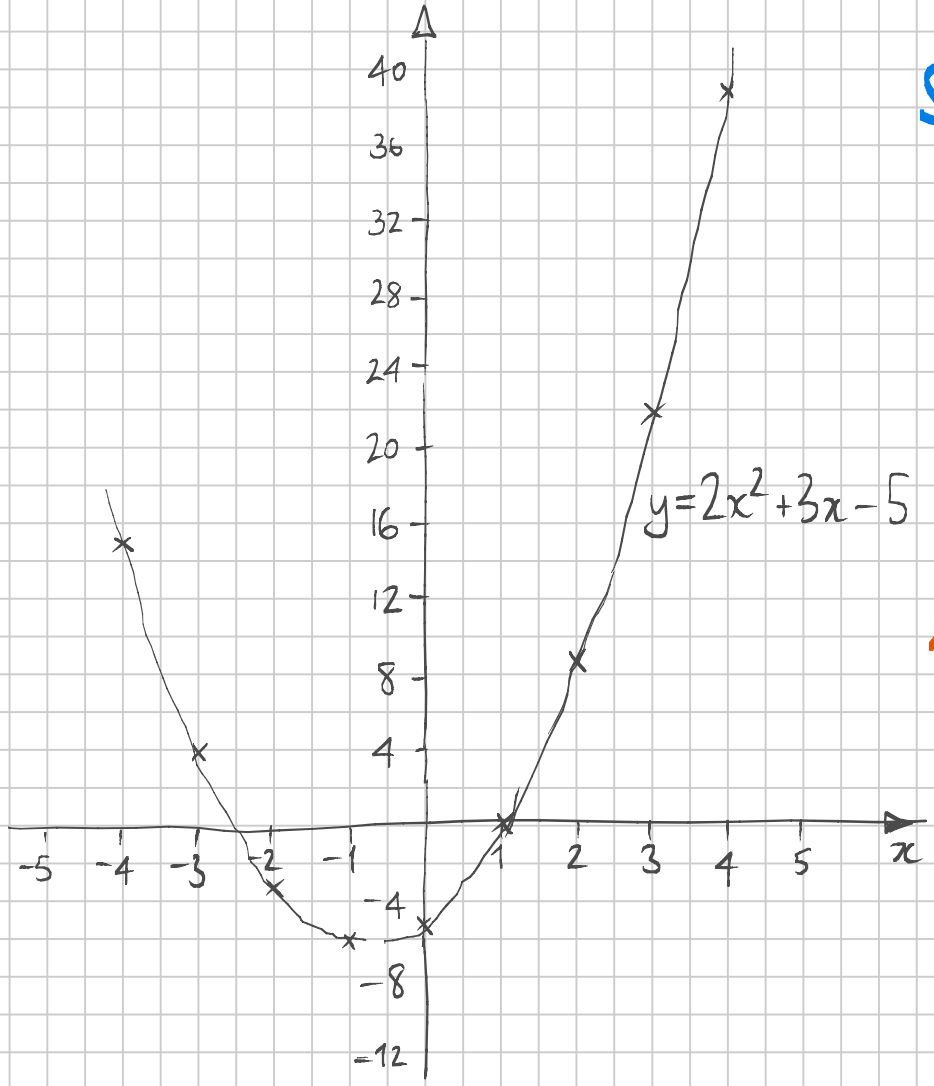
7a $y = 2x^2 + 3x - 5$

x	-4	-3	-2	-1	0	1	2	3	4
$2x^2$	32	18	8	2	0	2	8	18	32
$+3x$	-12	-9	-6	-3	+0	+3	+6	+9	+12
-5	-5	-5	-5	-5	-5	-5	-5	-5	-5
y	15	4	-3	-6	-5	0	9	22	39

Have you noticed (perhaps in horror!) that these numbers don't seem to be symmetrical anymore?

Just out of interest, can you still remember how to factorise things like $2x^2 + 3x - 5$?

76



You can? Good!

So $2x^2+3x-5 \equiv (2x\ \square)(x\ \square)$

$\equiv (2x+5)(x-1)$,

yes? ... remember that.

Luckily the graph, however, is symmetrical. But where is the line of symmetry?

This is no longer obvious.

Ok, it looks like it is around $x = -3/4$ $(x = -0.75)$

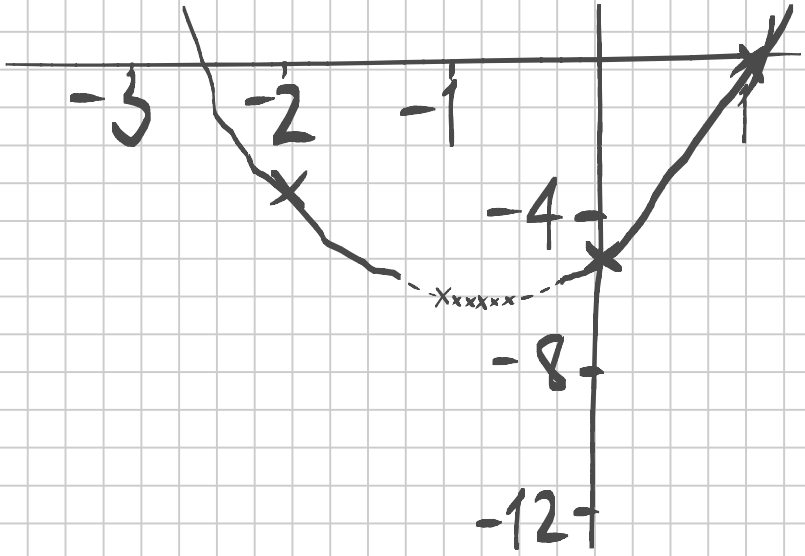
... but how can we be SURE?

Again you'll notice a gap on my graph that I need to fill...

7c Here's a bit more table:

x	-1	-0.9	-0.8	-0.75	-0.7	-0.6
$2x^2$	2	1.62	1.28	1.125	0.98	0.72
$+3x$	-3	-2.7	-2.4	-2.25	-2.1	-1.8
-5	-5	-5	-5	-5	-5	-5
y	-6	-6.08	-6.12	-6.125	-6.12	-6.08

And suddenly, the symmetry is clear... $x = -0.75$ IS our line of symmetry, AND the minimum value is -6.125.



You can see I've 'zoomed-in' on this bit to show what's going on.

I don't expect you to go into this detail yourselves, but it's nice to know you're right isn't it?

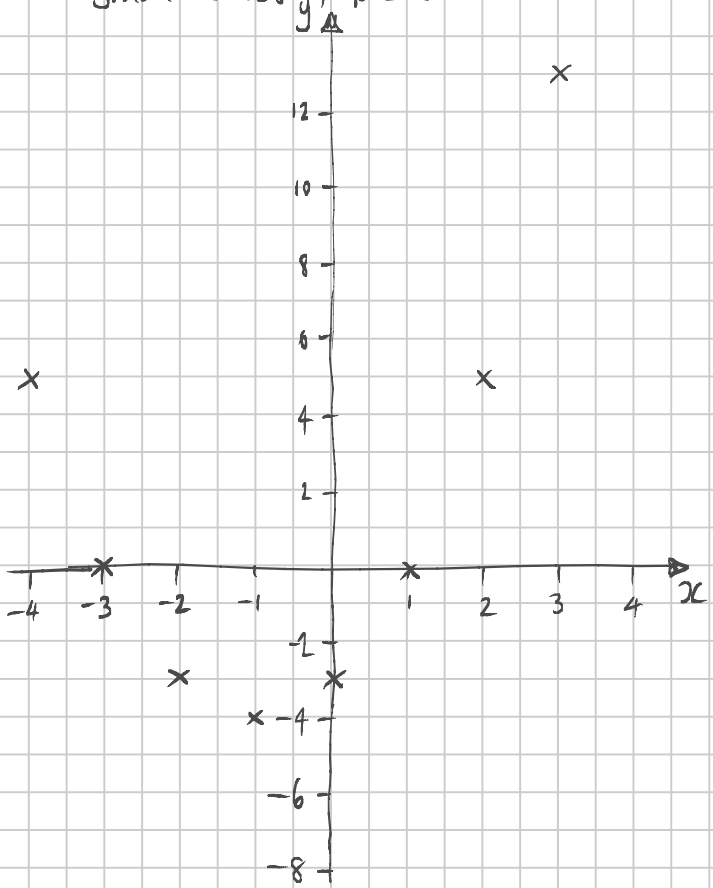
8a $y = x^2 + 2x - 6$

9a

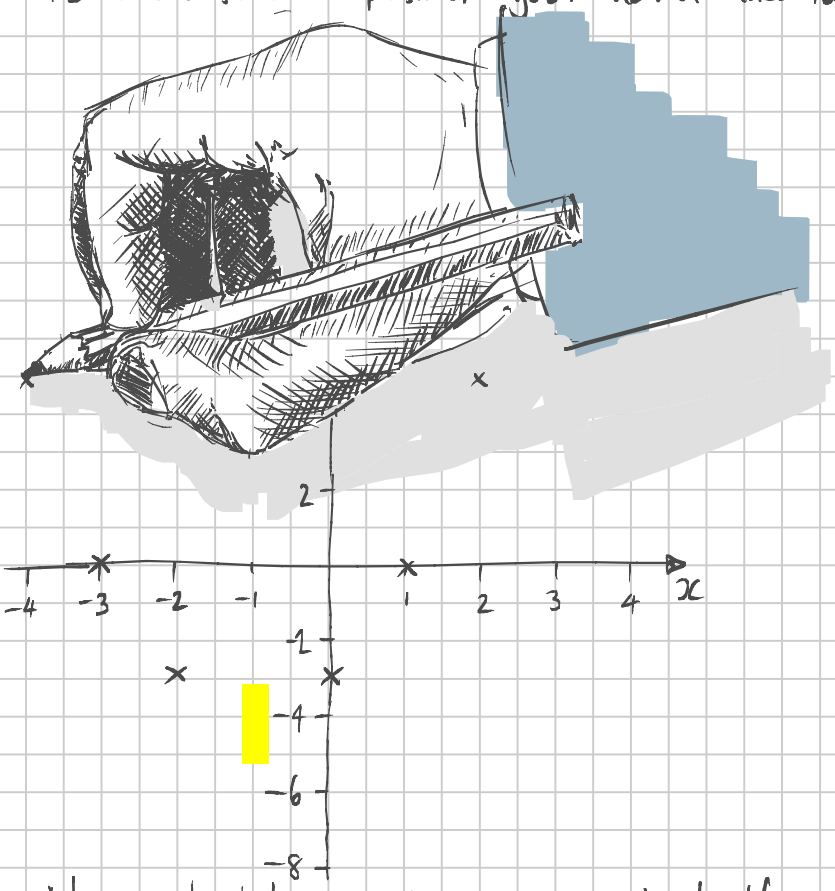
9b

How to draw a smooth quadratic curve

① Plot points with sharp pencil.
Small crosses, please.

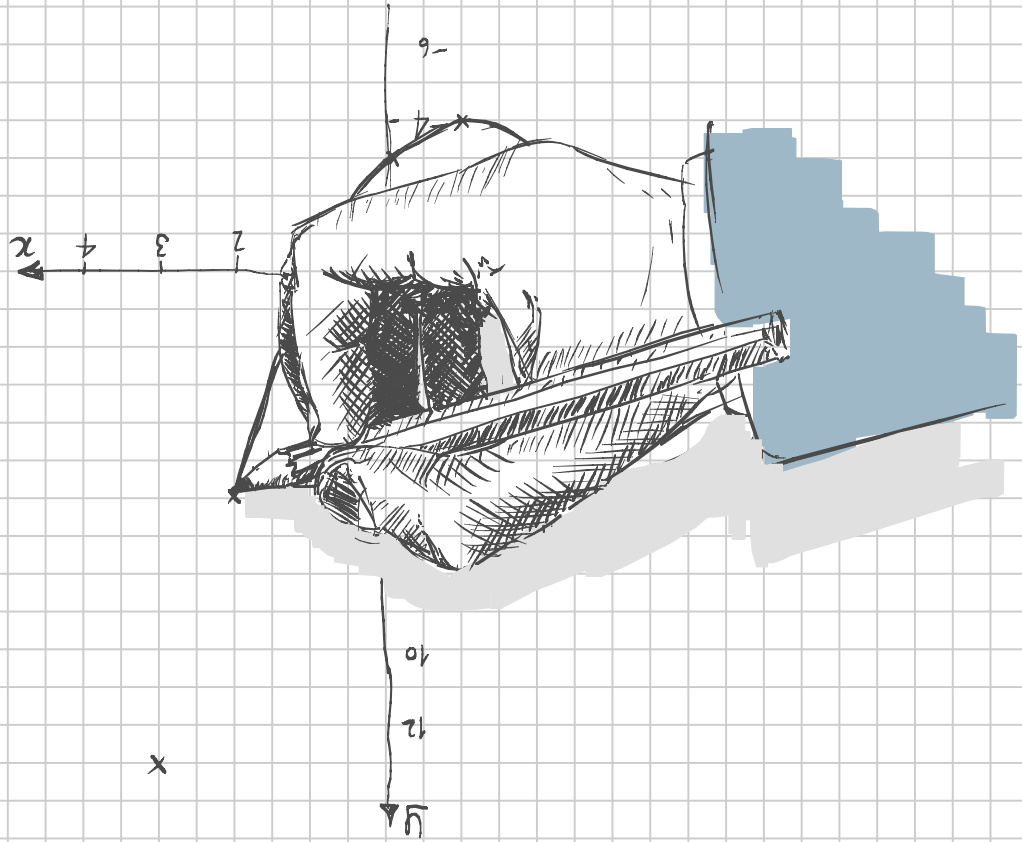


② Assuming you're right-handed turn the paper 45° clockwise and position your hand like this



Now sketch in the curve to halfway-rotate from your wrist rather than fingers if you can. Stop at the apex-here (-1, -4)

③ Now turn the paper upside down



Continue the curve from <u>within</u> the 'bowl' shape so that the curve continues to flow with your wrist, not against it.

Left-handers use the same method, but start from the right hand end of the curve and work back.